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Apéry limits of differential equations of order 4 and 5. (English summary)
Modular forms and string duality, 105–123, *Fields Inst. Commun.*, 54, *Amer. Math. Soc.*, *Providence, RI*, 2008.

The term "Apéry limit" in the title refers to the following procedure (of Apéry, which he used in 1978 to prove the irrationality of $\zeta(3)$): start with the recurrence

$$(n+2)^3A_{n+2}-(2n+3)(17n^2+51n+39)A_{n+1}+(n+1)^3A_n=0$$

with initial conditions $A_{-1}=0$ and $A_0=1$. Let B_n satisfy the same recurrence, but with initial conditions $B_0=0$ and $B_1=1$. Then $\lim_{n\to\infty}\frac{B_n}{A_n}=\frac{\zeta(3)}{6}$. The above recurrence translates into a linear differential equation of order 3 for the generating series $\sum_{n=0}^{\infty}A_nx^n$. In the present paper, the authors go systematically through their list [G. Almkvist et al., "Tables of Calabi–Yau equations", preprint, arxiv.org/abs/math/0507430] and investigate whether there are "Apéry limits" associated with Calabi-Yau differential equations of order 4 or 5. It turns out that for several cases such "Apéry limits" are already known, in some cases new limits are discovered and proved, several cases are listed where conjectural expressions for the limits have been found by the authors using the PSQL algorithm, and, finally, there are cases in which a limit could not be identified.

{For the entire collection see MR2478453 (2009i:11005)}

Reviewed by *Christian Krattenthaler*